Assignment 3

Textbook assignment: Chapter 4, "Limits and Differentiation," pages 4-1 through 4-34 and Chapter 5, "Derivatives," pages 5-1 through 5-38.

- A thorough understanding of functions and functional notation is necessary at this point. In mathematics a variable, y, is said to be a function of another variable, x. It is not enough to say that a change in x causes a corresponding change in y, because this is not always true; for example, when y equals a constant. To allow for certain exceptions, we have adopted the following definition: If for each value of variable x there is only one corresponding value y, then y is said to be a function of x. Consider the equation y = 3. This is a straight line parallel to the X axis and intersecting the Y axis at 3. No matter what value is assigned to x, there is a corresponding value for y; in this example 3.

Functional notation is used as a brief and convenient way of symbolizing that variable y is a function of variable x and is written $y = f(x)$ where y is the dependent variable and x is the independent variable. This is read, "y equals f of x." This means y is some function of x; that is, y has a value for each value of x. This symbolism does not mean that some quantity f is to be multiplied by some quantity x. It also does not indicate what the function of x is. It could be $1 - x^2$, $3 \sin x$, $x \log_{10} x$, or any function involving x as the independent variable.

If more than one function of x must be considered in a single discussion, a symbol other than f is used, such as: g(x), q(x), or h(x). For example, if it is given that y is a function of $x \cos x$, $0 \leq x < 5$, and $x^2 + 2$, $5 \leq x \leq 9$, this could be stated symbolically as

$$y = \begin{cases} f(x), & 0 \leq x < 5 \\ g(x), & 5 \leq x \leq 9 \end{cases}$$

where $f(x) = x \cos x$ and $g(x) = x^2 + 2$.

When using functional notation, we should remember that throughout a single problem or discussion, a given functional symbol such as f(x) will always represent the same function, and a symbol such as f(a) means that the function f(x) is to be evaluated at x = a. Stated in symbols, f(a) represents the value of f(x) at x = a.

As an example, find f(2) when

$$y = f(x) = x^2 - x + 2$$

To find f(2), we merely substitute a 2 every place an x occurs in f(x).

$$f(2) = 2^2 - 2 + 2 = 4$$

Likewise, f(x) may be evaluated for any constant value assigned to x and symbolized in functional notation by placing in parenthesis the value x will have for that particular problem. In the above example find f(-2), f(3), and f(a). Hence,

$$f(-2) = (-2)^2 - (-2) + 2 = 8$$

$$f(3) = 3^2 - 3 + 2 = 8$$

$$f(a) = a^2 - a + 2$$

Learning Objective:

Define "limit" and recognize techniques useful in evaluating a limit.

- In answering items 3-1 and 3-2, refer to the curve $y = x^2$.

3-1. y = 16 on the graph is considered to be the limit of

1. L as x is allowed to approach the value 4
2. f(x) as y is allowed to approach the value 4
3. f(x) as x is allowed to approach the value 4
4. $y = x^2$ as x is allowed to approach the value 16

3-2. If the curve is defined at x equals 4, it can then be said that the

1. limit of x = 4
2. $\lim\limits_{x \to 4} f(x) = f(4)$
3. limit of $x^2 = 16$
4. limit f(4) = 4

3-3. Which interval would NOT contain $\lim\limits_{x \to 5} x^2$?

1. 1 - 81
2. 16 - 36
3. 20.25 - 30.25
4. 4 - 6

3-4. Which statement below explains why L is the limit of f(x) as x approaches a?

1. The value of f(a) equals L
2. The function f(x), when evaluated at x equal to a, is such that f(x) - f(a) is equal to zero
3. The absolute value of the sum of f(x) and L is made larger than any test number δ by choosing an appropriate value for x very near, but not equal to, ε
4. The absolute value of the difference between f(x) and L can be made smaller than any other positive number named by choosing an appropriate value for x very near, but not equal to, a

3-5. If $\lim\limits_{x \to a} f(x) = L$, then f(a) exists.

1. True
2. False

- For all the functions given in items 3-6 through 3-8, the $\lim\limits_{x \to a} f(x)$ may be found by substituting a for x; that is, by evaluating f(a).

3-6. The limit of $3x^2 - 5 - x$ as x approaches -3 is

1. 10
2. 18
3. 25
4. 35

3-7. $\lim\limits_{x \to 2} (\sqrt{2x^2 - 2x} + 3x)$ is

1. 12
2. 8
3. $2(\sqrt{3} + 3)$
4. 4

3-8. Find $\lim\limits_{z \to a} f(z)$ when f(z) equals $\dfrac{3z^2 - z}{z - 1}$ and a equals 0.

1. 1
2. -1
3. 0
4. 5

- In items 3-9 through 3-12, find the limit of the functions and, where necessary, change the function by division or factoring so that an indeterminate form does not occur.

3-9. $\lim\limits_{t \to 2} 16t^2 + 20t - 30$ is

1. 0
2. 6
3. 36
4. 74

3-10. $\lim_{x \to 0} \frac{x^2 - 1}{x - 1}$ is

1. 1
2. 2
3. 0
4. x + 1

3-11. $\lim_{x \to \infty} \frac{x^4 - 2x^2 + 3}{2x^4 + x^3 - 3x^2 + x}$ is

1. 0
2. ∞
3. 3
4. $\frac{1}{2}$

3-12. $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$ is

1. 1
2. 2
3. 0
4. x + 1

3-13. Which of the following statements correctly describes the outcome of $\frac{0}{0}$ produced when you attempt to determine $\lim_{x \to a} f(x)$ by evaluating f(a)?

1. The limit is zero
2. The limit, if it exists, cannot be determined by this method because the outcome is in an indeterminate form
3. The limit is one
4. The limit is infinity

3-14. What is $\lim_{x \to 0} \frac{\sin x + x}{2}$, where x is in radians?

1. +1
2. $\frac{1}{2}$
3. -1
4. 0

- Items 3-15 through 3-19 relate to the limit theorems.

In answering items 3-15 through 3-18, select from column B the word statement of each of the theorems in column A.

A. THEOREMS

3-15. $\lim_{x \to a} f(x)\, g(x) = \left[\lim_{x \to a} f(x)\right]\left[\lim_{x \to a} g(x)\right]$

3-16. $\lim_{x \to a} [f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$

3-17. $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$, if $\lim_{x \to a} g(x) \neq 0$

3-18. $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

B. STATEMENTS

1. The limit of the quotient of two functions is equal to the quotient of their limits, provided the limit of the divisor is not equal to zero
2. The limit of the product of two functions is equal to the product of their limits
3. The limit of the sum of two functions is equal to the sum of the limits
4. The limit of a constant, c, times a function, f(x), is equal to the constant, c, times the limit of the function

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3-19. Using theorems 2 and 3 given in your text, find the limit of

$3(x - 2)(x^2 + 4)$ as x approaches 5.

1. 38
2. 81
3. 261
4. 340

Learning Objective:

Define and apply the concept of infinitesimals.

3-20. An infinitesimal results when a variable

1. approaches 0 as a limit
2. equals its limit
3. approaches ∞ as a limit
4. can be assigned a definite value

3-21. A variable approaches a constant as a limit if the difference between the variable and the constant becomes an infinitesimal.

1. True
2. False

● In items 3-22 through 3-26, mark your answer sheet True if the function becomes an infinitesimal in approaching its limit and False if it does not.

3-22. $\lim\limits_{x \to 3} \dfrac{x^2 + 5x - 6}{x + 6} - 2$

3-23. $\lim\limits_{x \to 4} (x^2 - 4x + 3)$

3-24. $\lim\limits_{x \to 2} (5x - x^3)$

3-25. $\lim\limits_{x \to -4} \dfrac{x^2 + 8x + 16}{x + 4}$

3-26. $\lim\limits_{x \to 0} \dfrac{x^4 + 2x^2 - 1}{x^2 - 3x + 2}$

3-27. Which statement concerning infinitesimals is FALSE?

1. The product of a constant and an infinitesimal is always an infinitesimal
2. The product of two or more infinitesimals is always an infinitesimal
3. The sum of two or more infinitesimals is always an infinitesimal
4. The ratio of two infinitesimals is always an infinitesimal

● A function is discontinuous at x equals a if the evaluation of f(a) causes division by zero. Since a point has no physical dimensions, the graph of a curve with only one missing point would appear to be a continuous curve, but mathematically it is considered to be discontinuous. The graph of the function $\dfrac{x^2 - 4}{x - 2}$ is discontinuous at x equals 2 because division by 0 occurs. This point is represented by a small circle drawn on the curve at x equals 2. Even though the curve does not exist at x equals 2, no gap would be left that could be physically seen. The small circle would actually contain an infinite number of points and is therefore only symbolic of the discontinuity.

Other forms of discontinuity can be graphically shown. The function, y = tan x, is continuous in the interval $-\frac{\pi}{2} < \tan x < \frac{\pi}{2}$, or $\frac{\pi}{2} < \tan x < \frac{3\pi}{2}$, but is discontinuous at these limits. This curve continues to repeat itself at π radian intervals and is discontinuous at $\pi/2 + n\pi$, where n is any integer.

Generally speaking, a function is continuous in a given interval if the curve is unbroken; that is, if it can be drawn in that interval without lifting the pen or pencil from the paper.

Learning Objective:

Define and recognize continuous functions.

3-28. At which one of the following values of x is the function $\frac{2}{x + 2} + \frac{4}{x^2 + x - 12}$ continuous?

1. -4
2. -2
3. 3
4. 4

3-29. Which of the following functions is defined at x equals 2?

1. $\frac{x}{x - 2}$
2. $\frac{\sqrt{13} - x}{x^3 - 8}$
3. $\frac{x^3 + 3x - 5}{x + 2}$
4. $\frac{1}{x^2 - 6x + 8}$

3-30. Which of the following functions is continuous at x equals c?

1. $\frac{a}{cx - c^2}$, $c \neq 0$
2. $\frac{2c - cx}{x - c}$, $c \neq 0$
3. $\frac{x^2 - c^2}{x + c}$, $c \neq 0$
4. $\frac{x^2 + c^2}{x^2 - c^2}$, $c \neq 0$

3-31. Which of the following functions of x is continuous for all values of x?

1. $h(x) = \frac{x^2 - 4}{x - 2}$
2. $g(x) = \begin{cases} \frac{x^2 - x - 6}{x - 3}, & x \neq 3 \\ 5, & x = 3 \end{cases}$
3. $q(x) = \begin{cases} \frac{x^2 - 3x - 4}{x + 1}, & x \neq 4 \\ 0, & x = 4 \end{cases}$
4. $f(x) = \begin{cases} \frac{x^3 - 8}{x - 2}, & x \neq -2 \\ 4, & x = -2 \end{cases}$

Learning Objective:

Use the limit concept in differentiating functions.

3-32. In the function $y = f(x)$, the limit of the ratio of Δy to Δx as Δx approaches zero is called the

1. integral of Δy
2. derivative of Δx
3. differential of y
4. derivative of f(x)

● Remember, the limit of an increment as it approaches zero is an infinitesimal and is considered to be zero. This does not, however, apply to the ratio of two infinitesimals. As pointed out in your text, the ratio may have any numerical value, including zero. Any term that includes an infinitesimal as a factor can be dropped because the whole term is an infinitesimal, except when two infinitesimals occur as a ratio. Note in the example below that when the limit is taken, terms containing increments that become infinitesimals drop out.

$$\lim_{\Delta x \to 0} (4x - 3x\Delta x + 1000\Delta x - \Delta x^3) = 4x$$

Here is an additional example using the four-step delta method given in your text. The derivative of

$y = 2x^2 - x + 8$ is found as follows:

If x were to increase by an increment Δx, the corresponding increase in y would be Δy. This is illustrated in the next equation where $y + \Delta y$ replaces the y and $x + \Delta x$ replaces all x's in the original equation.

$$y + \Delta y = 2(x + \Delta x)^2 - (x + \Delta x) + 8$$

Performing the indicated expansion of the squared binominal gives

$$y + \Delta y = 2(x^2 + 2x\Delta x + \Delta x^2) - (x + \Delta x) + 8$$

$$y + \Delta y = 2x^2 + 4x\Delta x + 2\Delta x^2 - x - \Delta x + 8$$

Now subtracting the original equation gives

$$\begin{array}{rl} y + \Delta y = & 2x^2 + 4x\Delta x + 2\Delta x^2 - x - \Delta x + 8 \\ y \quad\;\; = & 2x^2 \qquad\qquad\qquad\qquad\; - x \qquad\;\; + 8 \\ \hline \Delta y = & \qquad + 4x\Delta x + 2\Delta x^2 \qquad - \Delta x \end{array}$$

This step is allowable because equals are subtracted from equals. Next divide both sides of the equation by Δx.

$$\frac{\Delta y}{\Delta x} = \frac{4x\Delta x}{\Delta x} + \frac{2\Delta x^2}{\Delta x} - \frac{\Delta x}{\Delta x}$$

which reduces to

$$\frac{\Delta y}{\Delta x} = 4x + 2\Delta x - 1$$

Take the limit of both sides of the equation.

$$\lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} (4x + 2\Delta x - 1)$$

The left-hand member is symbolized $\frac{dy}{dx}$ and the right-hand member becomes $4x - 1$; thus $\frac{dy}{dx} = 4x - 1$.

● In items 3-33 through 3-35, find the derivative of the given function using the four-step delta method given in your text.

3-33. The derivative of $y = 3x^2 + 100$ is

1. $6x$
2. $6x^2$
3. $5x + 10$
4. $3x^2 + 6x\Delta x + 100$

3-34. The derivative of $y = 2x^3$ is

1. $2x^6$
2. $5x^3$
3. $6x^2$
4. $6x^2 + 6x$

3-35. The derivative of $y = \frac{1}{x^2}$ is

1. $2x$
2. $\frac{1}{2x}$
3. $-\frac{2}{x^3}$
4. $-\frac{1}{x} + 2$

● In answering items 3-36 and 3-37, find the slope of the tangent line to the given curve at the point indicated.

3-36. The slope of $y = 5x^2$ at $x = 4$ is

1. 5
2. 15
3. 30
4. 40

3-37. The slope of $y = x^2 - 4x + 4$ at $x = 1$ is

1. -2
2. 2
3. 0
4. 4

● Consider the graph of the curve $y = 5x^3 - 6x^2 - 3x + 3$.

The maximum or minimum values for this curve obviously are undefined since the curve grows without bounds. But in the interval between x equals -1 and x equals 2, the curve reaches a maximum at x equals $-\frac{1}{5}$ and a minimum at x equals 1.

It is these points which have physical significance and about which the text discussion is centered. These significant maxima and minima are usually referred to as local maxima and mimima to distinguish them from other kinds.

The text has defined the derivative to be equal to the slope of the tangent line on the curve at any point x in the interval under discussion. When the tangent line to the curve is parallel to the X axis, the value of the slope is zero. The tangent line can be parallel to the X axis (have a slope of zero) at only two points. These points are at x equals $-\frac{1}{5}$, where the curve is maximum, and at x equals 1, where the curve is minimum.

Because the derivative is equal to the slope of the tangent to the curve, maxima and minima can easily be found by determining the derivative, setting the derivative equal to zero, and determining the values of the independent variable that will make the derivative equal to zero. Since,

$$\frac{dy}{dx} = 15x^2 - 12x - 3$$

and

$$15x^2 - 12x - 3 = 0$$

then

$$3(5x^2 - 4x - 1) = 0$$

or

$$(5x^2 - 4x - 1) = 0$$

By factoring,

$$(5x + 1)(x - 1) = 0$$

such that

$$x = -\frac{1}{5} \text{ and } 1$$

The coordinates on the curve where a maximum or a minimum occurs can be found by substituting the determined values of x back into the original equation of the curve to find corresponding values of y. Thus when x = -1/5, then

$$y = 5\left(-\frac{1}{5}\right)^3 - 6\left(-\frac{1}{5}\right)^2 - 3\left(-\frac{1}{5}\right) + 3$$

$$= 3\frac{8}{25}$$

Therefore, the coordinates of the maxima are (-1/5,3 8/25). When x = 1

$$y = 5(1)^3 - 6(1)^2 - 3(1) + 3$$

$$= -1$$

Therefore, the coordinates of the minima are (1,-1).

- In answering items 3-38 and 3-39, determine the values of the independent variable for all maxima and minima of the given function.

3-38. There exists either a maximum or a minimum point on the curve

$y = 2x^2 - 8x$ at x equals

1. 1
2. 2
3. 8
4. 4

3-39. On the curve

$$s = \frac{1}{3}t^3 + \frac{1}{2}t^2 - 12t + 4,$$

where $-7 \leq t \leq 6$, there are either maximum or minimum points at t equals

1. 2 and -5
2. 3 and -4
3. 3 and 6
4. 3 and -2

Learning Objective:

Differentiate explicit and implicit functions through the use of rules and formulas.

- The formulas on the following page may be used for reference. In these formulas a, n, and C always represent a constant and u and v always represent a function of x.

Table 3A.--Formulas Involving Common Derivative Forms

1. $\frac{d}{dx}(a) = 0$

2. $\frac{d}{dx}(x) = 1$

3. $\frac{d}{dx}(x^n) = nx^{n-1}$

4. $\frac{d}{dx}(ax^n) = anx^{n-1}$

5. $\frac{d}{dx}(u + v) = \frac{du}{dx} + \frac{dv}{dx}$

6. $\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$

7. $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

8. $\frac{d}{dx}(au^n) = anu^{n-1}\frac{du}{dx}$

9. $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ (chain rule)

10. $\frac{dy}{dx} = \frac{1}{\frac{dx}{dy}}$

11. $\frac{d}{dx}(\sin u) = \cos u \frac{du}{dx}$

12. $\frac{d}{dx}(\cos u) = -\sin u \frac{du}{dx}$

13. $\frac{d}{dx}(\tan u) = \sec^2 u \frac{du}{dx}$

14. $\frac{d}{dx}(\cot u) = -\csc^2 u \frac{du}{dx}$

15. $\frac{d}{dx}(\sec u) = \sec u \tan u \frac{du}{dx}$

16. $\frac{d}{dx}(\csc u) = -\csc u \cot u \frac{du}{dx}$

17. $\frac{d}{dx}(\ln x) = \frac{1}{x}$

18. $\frac{d}{dx}(\ln u) = \frac{1}{u}\frac{du}{dx}$

19. $\frac{d}{dx}(e^x) = e^x$

20. $\frac{d}{dx}(e^u) = e^u \frac{du}{dx}$

21. $\frac{d}{dx}(a^x) = (\ln a)a^x$

22. $\frac{d}{dx}(a^u) = (\ln a)a^u \frac{du}{dx}$

3-40. The derivative of a constant is

1. one
2. zero
3. an infinitesimal
4. the constant

3-41. Given the function y = 12, as you move from point x to x + Δx, what happens to the value of y + Δy?

1. It increases
2. It decreases
3. It varies in proportion to Δx
4. It remains the same

3-42. The derivative of $f(x) = x^4$ is

1. $\frac{x^3}{4}$
2. x^3
3. $4x^2$
4. $4x^3$

3-43. If $f(x) = x^n$ and

$$\frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(x + \Delta x)^n - x^n}{\Delta x},$$

then as Δx approaches zero, $f'(x)$ equals

1. $\frac{x^{n-1}}{0}$
2. $\frac{x^{n-1}}{\Delta x}$
3. nx^{n-1}
4. $\frac{x + \Delta x}{\Delta x}$

3-44. $\frac{dy}{dx}$ of $y = kx^2$ is

1. $2x$
2. $2kx$
3. kx
4. $\frac{k}{x}$

3-45. The derivative of the sum of two or more differentiable functions of x is equal to the

1. product of their derivatives
2. product of their limits
3. sum of their limits
4. sum of their derivatives

3-46. Find $\frac{dy}{dx}$ of $y = tx^4 + sx^3 + x^2 + 6$.

1. $4x^3 + 3x^2 + 2x + 1$
2. $4tx^3 + 3sx^2 + 2x$
3. $4tx^3 + 3sc^2 + 2x + 1$
4. $4x^3 + 3x^2 + 2x + 6$

- In answering items 3-47 and 3-48, refer to the following information:

The relationship between the time, t, of flight in seconds and the altitude, a, in feet of a projectile is given approximately by the formula $a = vt - 16t^2$, where v is the muzzle velocity of the projectile. Assuming that a gun having a muzzle velocity of 384 feet per second is fired, the formula for the projectile's altitude becomes $a = 384t - 16t^2$.

3-47. How long after the gun is fired will the altitude be maximum? (Hint: Solve for $\frac{da}{dt}$ and use the procedure for determining a maximum value.)

1. 14 seconds
2. 12 seconds
3. 10 seconds
4. 8 seconds

3-48. What will be the maximum altitude reached by the projectile?

1. 1,152 feet
2. 2,304 feet
3. 3,456 feet
4. 4,608 feet

3-49. The derivative of the product of two differentiable functions of x equals the product of the derivatives of the functions.

1. True
2. False

- In answering items 3-50 through 3-52, employ the theorem for finding the derivative of a product.

3-50. Find $\frac{dy}{dx}$ of $y = x^2(3x)$.

1. $6x^2$
2. $6x^2 + 3x$
3. $3x^2 + 6x$
4. $9x^2$

3-51. If $f(x) = (x^2 + 3x)(x^3 + 4)$, what is $\frac{dy}{dx}$?

1. $6x^3 + 9x^2$
2. $5x^4 + 12x^3 + 8x + 12$
3. $(x^2 + 3x)(3x^2 + 1) + (x^3 + 4)(2x)$
4. $(x^2 + 3x)(3x^2 + 1) + (x^3 + 4)(2x + 3)$

3-52. $\frac{dy}{dx}$ of $3x^2(2x + 4)(tx^3)$ is

1. $(2x + 4)(tx^3)(6x) + (3x^2)(tx^3)(6) + (3x^2)(2x + 4)(3tx^2)$
2. $(3x^2)(2x + 4)(3x^2) + (2x + 4)(3tx^2)(2) + (3x^2)(tx^3)(6x)$
3. $(3x^2)(2x + 4)(3tx^2) + (2x + 4)(tx^3)(2) + (3x^2)(tx^3)(2)$
4. $(3x^2)(2x + 4)(3tx^2) + (2x + 4)(tx^3)(6x) + (3x^2)(tx^3)(2)$

3-53. If $w = g(x)$ and $z = h(x)$, then the derivative of $\frac{z}{w}$ equals

1. $\frac{wg'(x) - zh'(x)}{z^2}$
2. $\frac{wh'(x) - zg'(x)}{w^2}$
3. $\frac{zg'(x) - wh'(x)}{z^2}$
4. $\frac{zh'(x) - wg'(x)}{w^2}$

3-54. The derivative of

$$f(x) = \frac{x^2 - 3x + 3}{x^3 + 2} \text{ is}$$

1. $\frac{-x^4 + 6x^3 + 9x^2 + 6x + 3}{(x^2 + 2)^2}$
2. $\frac{x^4 + 6x^3 + 9x^2 + 6x - 3}{(x^3 + 2)^2}$
3. $\frac{-x^4 + 6x^3 - 9x^2 + 4x - 6}{(x^3 + 2)^2}$
4. $\frac{x^4 + 9x^3 - 9x^2 + 4x - 6}{(x^3 + 2)^2}$

3-55. If $y = [u(x)]^t$, then $\frac{dy}{dx}$ equals

1. $t[u(x)]^{t-1}\,[u'(x)]$
2. $t[u(x)]^{t-1}\,\frac{u'(x)}{t'(x)}$
3. $t[u(x)]^{t-1}$
4. $\frac{tu'(x)}{[u(x)]^{t-1}}$

3-56. What is the derivative of $g(x) = (x^4 + 3x^3 + 2x)^5$?

1. $5(x^4 + 3x^3 + 2x)(4x^3 + 9x + 2)$
2. $5(x^4 + 3x^3 + 2x)(4x^3 + 9x^2 + 2)$
3. $5(x^4 + 3x^3 + 2x)^4(4x^3 + 9x + 2)$
4. $5(x^4 + 3x^3 + 2x)^4(4x^3 + 9x^2 + 2)$

3-57. Express the function

$$f(x) = \frac{1}{\sqrt{(x + 3)^3}}$$

using fractional exponents.

1. $(x + 3)^{-3/2}$
2. $(x + 3)^{3/2}$
3. $\frac{1}{(x + 3)^{-1/2}}$
4. $\frac{1}{(x + 3)^{-3/2}}$

3-58. $\frac{dy}{dx}$ of $f(x) = \frac{4x - 6}{\sqrt{x^2 + 3x + 2}}$ equals

1. $(4x^2 - 9)\sqrt{x^2 + 3x + 2} + 4\sqrt{x^2 + 3x + 2}$
2. $\frac{9 - 4x^2}{\sqrt{x^2 + 3x + 2}} + \frac{4}{\sqrt{x^2 + 3x + 2}}$
3. $\frac{4}{\sqrt{x^2 + 3x + 2}} - \frac{4x^2 - 9}{\sqrt{(x^2 + 3x + 2)^3}}$
4. $\frac{4x^2 - 9}{\sqrt{x^2 + 3x + 2}} + 4\sqrt{x^2 + 3x + 2}$

3-59. If $y = (z + 3)^2$ and $z = x + 3$, then according to the chain rule, $\frac{dy}{dx}$ equals

1. $\frac{dy}{du}\,\frac{du}{dx}$
2. $\frac{dy}{dx}\,\frac{dx}{dz}$
3. $\frac{dy}{dz}\,\frac{dz}{dx}$
4. $\frac{dz}{dx}\,\frac{dy}{dx}$

3-60. If $y = (t^2 + 1)^3$ and $t = (x^2 + 5)$, then $\frac{dy}{dx}$ equals

1. $12x(t^2 + 1)^2$
2. $12(x^2 + 5)[(x^2 + 5)^2 + 1]^2$
3. $3[(x^2 + 5)^2 + 1]^2$
4. $12x(x^2 + 5)[(x^2 + 5)^2 + 1]^2$

3-61. The derivative of an inverse function is equal to

1. the negative reciprocal of the derivative of the direct function
2. the reciprocal of the derivative of the direct function
3. the derivative of the direct function
4. the negative of the derivative of the direct function

3-62. If $x = \frac{5}{y} + y^2$, then $\frac{dy}{dx}$ equals

1. $y^2(2y^3 - 5y)^{-1}$
2. $y^2(2u^3 - 5)^{-2}$
3. $\frac{y^2}{2y^3 - 5}$
4. $\frac{y^2}{2y^3 - 5y}$

3-63. Which of the following statements is TRUE regarding the expression $ax^2 + xy + y^2 = 0$?

1. x is an explicit function of y
2. y is independent of x
3. y is an explicit function of x
4. y is an implicit function of x

3-64. The derivative $\frac{dy}{dx}$ of the function $x^2 + xy^2 + y = 0$ is

1. $\frac{-2x - y^2}{1 + 2x}$
2. $\frac{-2x - y^2}{1 + 2xy}$
3. $\frac{2x + y^2}{1 + 2x}$
4. $\frac{2x + y^2}{1 + 2xy}$

● The expression $\frac{d}{dx}(x^2)$ means to take the derivative of the quantity within the parentheses with respect to x; in this case $2x \frac{dx}{dx}$ or simply $2x$ because $\frac{dx}{dx}$ is 1.

Another example would be $\frac{d}{dz}(x^2 + 2z + 5)$, which means to take the derivative of the term $x^2 + 2z + 5$ with respect to z, or $2x \frac{dx}{dz} + 2$. Since the ratio $\frac{dx}{dz}$ is unknown, it is necessary to retain it as a factor.

3-65. Find $\frac{d}{dx}(\cos \theta)$.

1. $\sin \theta \frac{d\theta}{dx}$
2. $-\sin \theta \frac{d\theta}{dx}$
3. $-\cos \theta \frac{d\theta}{dx}$
4. $\cos \theta \frac{d\theta}{dx}$

● When differentiating trigonometric functions, it is important to remember to find the derivative of the angle in addition to the derivative of the function. The following example illustrates this point:

$\frac{dy}{d\theta}$ of $y = \cos 4\theta$ equals

$$\frac{d}{d\theta}(\cos 4\theta) = -\sin 4\theta \frac{d}{d\theta}(4\theta)$$
$$= -\sin 4\theta\ (4) \frac{d\theta}{d\theta}$$
$$= -4 \sin 4\theta$$

When using trigonometric functions raised to powers, you first apply the power rule for differentiating, then you apply the rules for differentiating the function, and finally you differentiate the angle. (PFA--power, function, angle --in that order.)

For example, if $y = (\tan 4\theta)^2$, then you determine $\frac{dy}{d\theta}$ by first applying the power rule,

$$\frac{dy}{d\theta} = 2(\tan 4\theta)\ \frac{d}{d\theta}(\tan 4\theta)$$

Next you apply the rule for differentiating tan 4θ,

$$\frac{dy}{d\theta} = 2(\tan 4\theta)(\sec^2 4\theta)\ \frac{d}{d\theta}(4\theta)$$

Finally, you differentiate the angle 4θ,

$$\frac{dy}{d\theta} = 2(\tan 4\theta)(\sec^2 4\theta)(4)$$

or

$$\frac{dy}{d\theta} = 8 \tan 4\theta \sec^2 4\theta$$

3-66. $\frac{d}{dx}\left(\frac{\sin u}{\cos u}\right)$ is equivalent to

1. $\frac{1}{\sec^2 u}\ \frac{du}{dx}$
2. $\frac{(\cos^2 u + \sin^2 u)}{\sec^2 u}\ \frac{du}{dx}$
3. $\frac{\cos^2 u \frac{du}{dx} - \sin^2 u \frac{du}{dx}}{\cos^2 u}$
4. $\frac{(\cos u)(\cos u)\ \frac{du}{dx}}{\cos^2 u} - \frac{(\sin u)(-\sin u)\ \frac{du}{dx}}{\cos^2 u}$

3-67. The function $y = \sin 2x \cos x$ has a derivative of

1. $-\sin^2 x + 2 \cos^2 x$
2. $-\sin 2x \sin x + \cos x \cos 2x$
3. $-\sin 2x \sin x + 2 \cos x \cos 2x$
4. $\sin 2x \sin x + 2 \cos x \cos 2x$

3-68. If $y = (\tan 2\theta)^2(\sin 2\theta)$, then $\frac{dy}{d\theta}$ equals

1. $(\tan^2 2\theta)(\cos 2\theta)$ $+ (\sin 2\theta)(2)(\tan 2\theta)(2)(\sec^2 2\theta)$
2. $(\tan^2 2\theta)(2)(\cos 2\theta)$ $+ (\sin 2\theta)(2 \tan 2\theta)(\sec^2 2\theta)$
3. $(\tan 2\theta)^2(\cos 2\theta)$ $+ (\sin 2\theta)(2)(\tan 2\theta)(\sec^2 2\theta)$
4. $(\tan^2 2\theta)(\cos 2\theta)(2)$ $+ (\sin 2\theta)(2)(\tan 2\theta)(\sec^2 2\theta)(2)$

3-69. Find $\frac{dy}{dx}$ of $y = e^{2x} + 1$.

1. $2e^{2x} + 1$
2. $e^{2x} + 1$
3. $2e^{2x}$
4. $2xe^{2x}$

3-70. Find $\frac{dy}{dx}$ of $y = 8^{4x} + \ln (x^4)$.

1. $8^{4x} + \frac{1}{x^4}$
2. $4(\ln 8)8^{4x} + \frac{4}{x}$
3. $(\ln 8)8^{4x} + \frac{4}{x}$
4. $4(\ln 8)8^{4x} + 4x^3$